Logarithmic relaxations in a random-field lattice gas subject to gravity

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A simple lattice-gas model with random fields and gravity is introduced to describe a system of grains moving in a disordered environment. Off-equilibrium relaxations of bulk density and its two time correlation functions are numerically found to show logarithmic time dependences and ''aging'' effects. Similarities with dry granular media are stressed. The connections with off-equilibrium dynamics in other kinds of ''frustrated'' lattice models in the presence of a directional driving force (gravity) are discussed to single out the appearance of universal features in the relaxation process. [S1063-651X(99)02903-7]

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I. INTRODUCTION

Relaxation properties of granular media in the presence of low amplitude vibrations are dominated by steric hindrance, friction, and inelasticity. They typically show very long characteristic times [1]. For instance, the grain compaction process in a box, i.e., the increase of bulk density in the presence of gentle shaking, seems to follow a logarithmic law [2–6].

These very slow dynamics resemble those of glassy systems [7–9]. Moreover, these materials also show ''aging'' and ''memory'' effects [2,10] with logarithmic scaling and metastability [2,10]. The presence of reversible-irreversible cycles (along which one can identify a sort of ''glassy transition'') as well as a dependence on ''cooling'' rates enhance the similarities with glassy systems. However, in granular media, as soon as external forces are cut off, grains come rapidly to rest. The microscopic origin of motion is thus quite different from thermal systems (glasses, magnets) where random particle dynamics is ensured by temperature. On this basis it is rather surprising to find those similar behaviors in off-equilibrium dynamics.

Based on this resemblance, frustrated lattice-gas models have been recently introduced to describe these slow logarithmic processes found in granular matter (where the shaking amplitude plays the role of an ''effective'' temperature of the system) [3,6].

The aim of the present paper is twofold. On one hand, we describe the correspondences with experiments on granular matter of some dynamical behaviors observed in a frustrated lattice-gas model subject to gravity; on the other hand, we emphasize the appearance of some very general features in the off-equilibrium dynamics in the presence of gravity in a broad class of frustrated lattice-gas models. At first, we thus introduce a very simple random field model to describe a system of grains moving in a disordered environment. The relaxation properties of the model are then studied via Monte Carlo simulations. We stress the relations with granular media and the implications of our results for the understanding of the behaviors of such materials. Finally, the connections with others kinds of ''frustrated'' models are discussed to single out the universal features of this class of lattice systems.

II. THE MODEL

Along the lines of the Ising frustrated lattice gas (IFLG) and TETRIS models [3,6], we consider a system of particles moving on a square lattice tilted by 45°. Because of the hard-core repulsion, every site of the lattice cannot be occupied by more than one particle. Moreover, the system is dilute, so that there will be empty sites on the lattice.

Although in real systems particles have a rich variety of shapes, dimensions, and orientations in space, for the sake of simplicity we restrict ourselves here to the case of one type of grain with an elongated form (as in the TETRIS model [6]) which can lie on the lattice with two different orientations (Fig. 1). Each particle is thus characterized by an intrinsic degree of freedom which is its orientation in space. It can have only two possible values corresponding to the two possible positions of the particle on the lattice sites. Since particles cannot overlap, particles with the same orientation can-

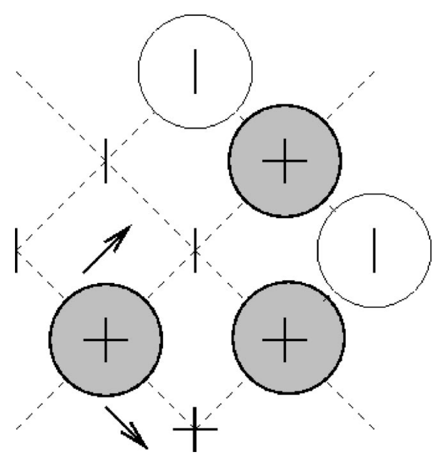

FIG. 1. Schematic picture of the random field frustrated Ising model. Plus and minus signs represent the orientation of the random field on each site of the lattice. Filled circles represent the particles with the two possible orientations: $S_i=-1$ (white) and $S_i=+1$ (gray). The coupling between nearest-neighbor (n.n.) spins is always antiferromagnetic (dashed lines). Only gray particles can stay on *plus* sites and only white particles can occupy *minus* sites. The result is that each particle on a site of the lattice has to fit either the geometry imposed by the field or the geometry of the n.n. spins.

not occupy two nearest-neighboring sites on the lattice. As a result, a local geometrical constraint is generated in the dynamics.

In real granular material each grain moves in a disordered environment made of the rest of the granular systems itself. To schematically describe such a situation without engaging the full complexity of the problem, we consider the grains of our model immersed in a disordered media whose disorder we presume to be "quenched." This choice may correspond, more strictly, to the case of grain motion on a geometrically disordered substrate such as a box with geometric asperities on its surface. At this stage, the physical feature to embody is the restriction in the grain motion produced by the environment disorder. Therefore, in our model a particle can occupy a lattice site only if its orientation fits both the local geometry of the medium (Fig. 1) and the geometrical interaction with the closely neighboring particles.

These ideas can easily be formalized using Ising spins in a magnetic language. A spin S_i (with $S_i = \pm 1$) can be identified with the internal degree of freedom which characterizes the twofold orientation of particle i. The "geometrical" interaction between nearest-neighboring grains, which must be antiparallel (to be nonoverlapping neighbors), is realized with antiferromagnetic couplings (of infinite strength) between nearest neighbors [11]. Analogously, the random geometry of the environment, which forces a grain to have a definite orientation to fit the local geometry, may be described as a strong random magnetic field attached to each site of the lattice.

The Hamiltonian of this random field Ising system with vacancies, in the presence of gravity, can be thus written as

$$\mathcal{H} = J \sum_{\langle ij \rangle} (S_i S_j + 1) n_i n_j - H \sum_i h_i S_i n_i + g \sum{}_i n_i y_i, \quad (1)$$

where h_i are random fields, i.e., quenched variables which assume the values ± 1, and n_i is the occupancy variable: $n_i = 1$ if site i is occupied by a particle, $n_i = 0$ otherwise. Here g is the gravity constant and y_i corresponds to the height of the site i with respect to the bottom of the box (grain mass is set to unity). J and H represent the repulsions felt by particles if they have a wrong reciprocal orientation or if they do not fit the local geometry imposed by the random fields, respectively. Here we study the case in which $J = H = \infty$, i.e., the case in which the geometric constraints are infinitely strong. In this case there will always be sites where "spins" cannot fulfill simultaneously every constraint, and thus will remain vacant.

III. THE TAPPING

In a tapping experiment on granular media, a dynamic is imposed on the grains by vibrations characterized by the shaking normalized intensity Γ (Γ is the ratio of the shake peak acceleration to the gravity acceleration g; see [2]). In the regime of low shaking amplitudes (small Γ), the presence of dissipation dominates grain dynamics, and in a first approximation the effects of inertia on particle motion may be neglected. To embody this scheme in our model we introduce two-step diffusive Monte Carlo dynamics for the particles.

The dynamics is very simple with particles moving on the lattice either upwards or downwards with respective probabilities p_{up} and p_{down} (with $p_{\mathrm{down}} = 1 - p_{\mathrm{up}}$).

In the first step, vibrations are on with $p_{\mathrm{up}} \neq 0$. Particles can then diffuse for a time τ_0 in any direction yet with the inequality $p_{\mathrm{up}} < p_{\mathrm{down}}$, and always preserving the above local geometric constraints. In the second step, vibrations are switched off and the presence of gravity imposes $p_{\mathrm{up}} = 0$. Particles can then move only downwards. In both steps particle orientation (i.e., its spin S_i) can flip with probability 1 if there is no violation of the above constraints, and does not flip otherwise.

In this single tap two-step dynamics, we let the system reach a *static* configuration, i.e., a configuration in which particles cannot move anymore. In our Monte Carlo tapping experiment a sequence of such a vibration is applied to the system.

Under tapping, a granular system can move in the space of available microscopic configurations in a way similar to that in which thermal systems explore their phase space. The key parameter of the tapping dynamics is thus the ratio $x = p_{\mathrm{up}} / p_{\mathrm{down}}$, which is linked to the experimental *amplitude* of shaking. An effective "temperature," T, can thus be introduced for the above Hamiltonian with $x \equiv e^{-2g/T}$, which in turn relates to granular media real shakes (see [1]) via the equality $\Gamma^a \sim T/2g \equiv 1/\ln(1/x)$, with $a \sim 1,2$ [1].

IV. COMPACTION

In order to investigate the dynamical properties of the system when subjected to vibrations, we study the behavior of two basic observables which are the density and the density-density time-dependent correlation function. The latter is considered in the next section.

The system is initialized filling the container by randomly pouring grains at the top, one after the other. Particles then fall down subject only to gravity, always preserving the model local geometric constraints. Once they cannot move down any longer, they just stop. From this loose packing condition, the system is then shaken by a sequence of vibrations of amplitude $x = p_{\mathrm{up}} / p_{\mathrm{down}}$ with the two-step diffusive dynamics described above.

Specifically, we have studied a 2D square lattice of size 30×60 (the results have been observed to be robust for system size changes). The lattice has periodic boundary conditions in the horizontal direction and a rigid wall at its bottom. The particle motion is thus occurring on a cylinder.

After each vibration t_n (where t_n is the nth "tap") the bulk density is measured, i.e., the density $\rho(x, t_n)$ in the lower 25% of the box.

Results for the compaction process are shown in Fig. 2. Different curves correspond to different values of the amplitude x, which ranges from $x = 0.001$ up to $x = 0.2$. Data are averaged over 10 different initial conditions and, for each initial condition, over 10 random field configurations. The duration of each tap was kept fixed to $\tau_0 = 30$ (time is measured in terms of per particle Monte Carlo step) in all the simulations. We have also checked that the qualitative general features exhibited by the model do not depend substantially on the choice of this value.

In analogy to experimental results, the value of x is a crucial parameter which controls the dynamics of the com-

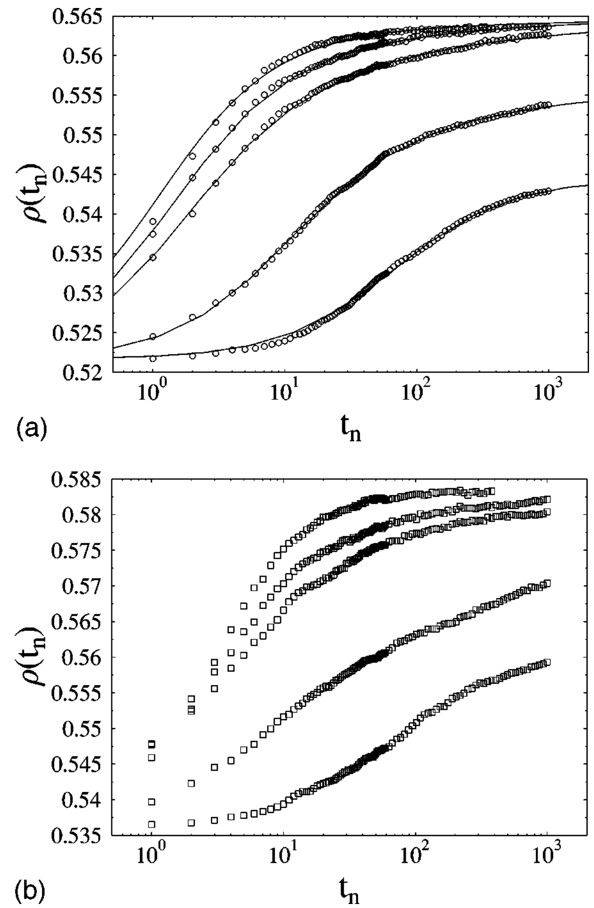

FIG. 2. (a) Bulk density (measured in the lower 25% of the box) relaxation for the tap's amplitude $x=0.001,0.01,0.05,0.1,0.2$ and the tap's duration $\tau=30$. Averages are taken over 10 different initial conditions and for each initial condition over 10 random field realizations. The solid line is a least square fit to the inverse logarithmic form discussed in the text. (b) As in (a), relaxation of the density measured in the region on the bottom between 25% and 50% in the height of the system. We find a behavior analogous to that described in (a).

paction process as well as the final static packing density [2]. In qualitative agreement with experimental findings, we observe that the stronger the shaking is (i.e., the higher x or T is), the faster the system reaches a higher packing density, as shown in Fig. 2. However, this is in contrast with the intuitive expectation about simple systems, such as (lattice) gases in the presence of gravity, where higher temperatures correspond to lower equilibrium densities [14,15].

It is known experimentally that, by gently shaking a granular system, the density at the bottom of the box increases very slowly until it reaches its equilibrium value. The best fit for the density relaxation is an inverse logarithmic form of the type [2],

$$\rho(t_n)=\rho_\infty-\Delta\rho_\infty\frac{\ln A}{\ln\left(\dfrac{t_n}{\tau}+A\right)}, \tag{2}$$

where ρ_∞ is the asymptotic density, $\Delta\rho_\infty=\rho_\infty-\rho_i$ is the difference between the asymptotic and initially measured density, $\rho_i\equiv\rho(t_n=0)\simeq 0.522$, and A and τ are two fitting parameters. Experimentally, $\rho_\infty,\Delta\rho_\infty,A$, and τ depend on the value of the normalized shaking amplitude Γ.

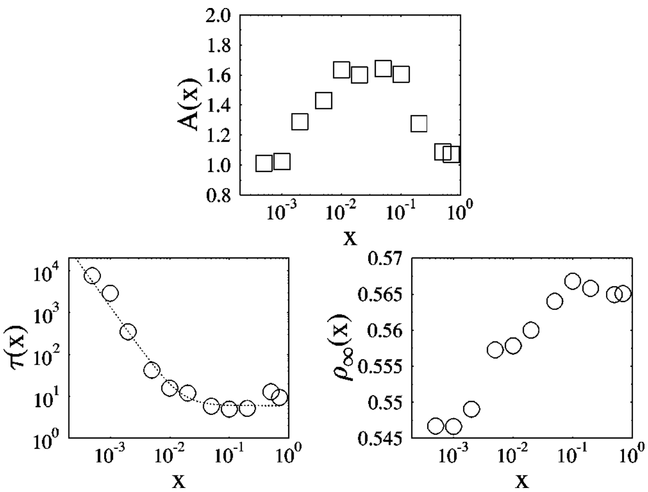

FIG. 3. Density relaxation fitting parameter A, τ, and ρ_∞ as a function of the intensity of vibrations x. In correspondence with experimental findings, ρ_∞ increases with increasing x and, above $\tilde{x}$, it becomes almost constant. Such a behavior is in contrast with the intuitive expectation about simple lattice gases in the presence of gravity, where lower equilibrium densities should correspond to higher temperatures.

Our different Monte Carlo relaxation curves for the density, obtained for different intensity of vibration x, show a behavior very similar to the experimental findings and can be well fitted with the inverse logarithmic law (2). As stated, the amplitude of shaking x determines the fitting parameters (ρ_∞,A, and τ) and their behavior seems to indicate a smooth crossover between two different regimes at varying x.

Figure 3 shows the relaxation of τ to a constant value with increasing x. The analytical dependence on x can be fitted with a power-law form of the type

$$\tau\sim\left(B+\frac{1}{x^\gamma}\right). \tag{3}$$

τ can be interpreted as the minimum time over which one starts to observe a compaction in the system. For small x, τ exhibits an algebraic dependence on x with $\gamma=2.0$, while when x crosses a certain threshold, say $\tilde{x}\sim 0.1$, it saturates to a constant value which is independent of the tapping amplitude.

The behavior of the final packing density of the system, ρ_∞, is also shown in Fig. 3. It increases with increasing x and then, when the shaking intensity is greater than the typical value $\tilde{x}$, it becomes almost constant up to the x value we considered. These results are consistent with those found in Ref. [3], and are in qualitative agreement with the above described experimental findings.

It is more difficult to distinguish a definite trend for the value of the parameter A versus x (see Fig. 3). However, it shows a change of behavior in correspondence with a definite tapping amplitude which still has approximately the same value $\tilde{x}$.

The recorded behavior of ρ_∞ with x (or Γ in the experiments), which, as stated, is in contrast with the expected

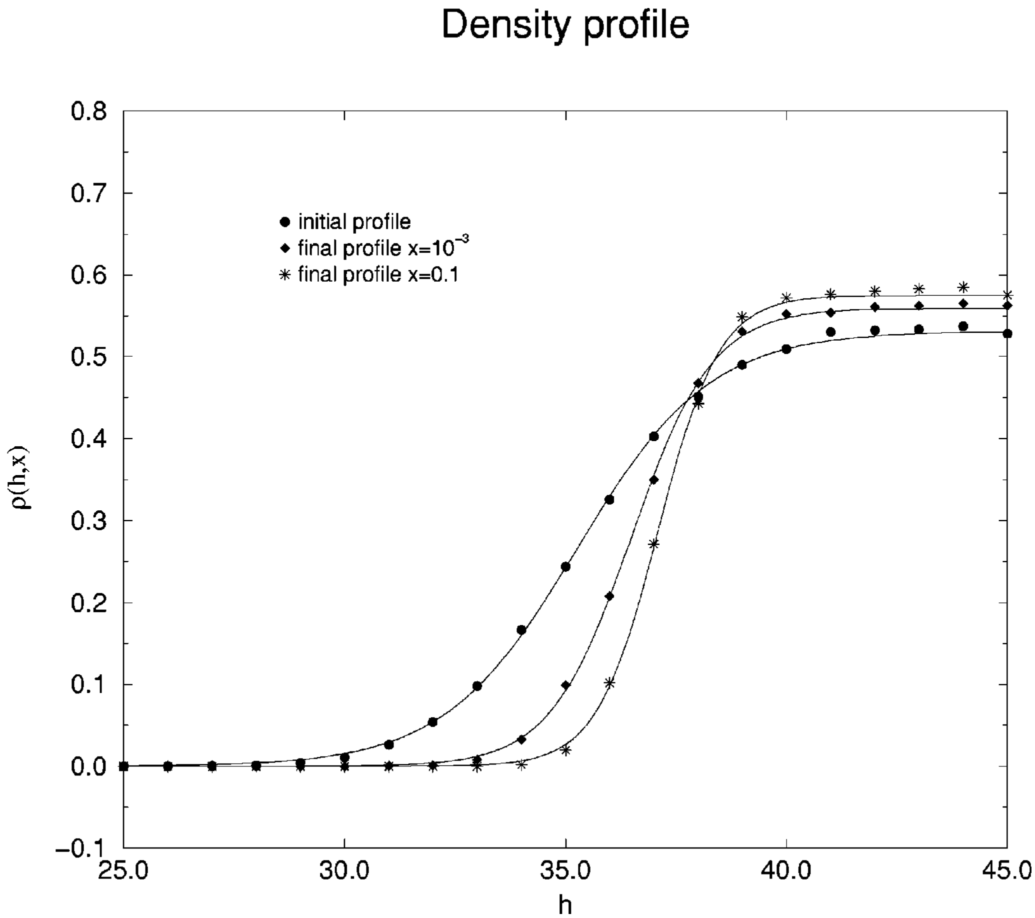

FIG. 4. Density profile with increasing depth from the top of the box. Different curves correspond to common initial configuration, and the final profile for a shaking amplitude $x=0.001$ and $x=0.1$. Superimposed are the Fermi-Dirac fits described in the text.

equilibrium values, and the observed rough crossover between two regions, suggest that, on typical time scales of both our Monte Carlo and real experiments, one is typically still far from equilibrium. Actually, if x is sufficiently small, the granular system has very long characteristic times, as shown by the presence of logarithmic relaxations. We will address the question of the out-of-equilibrium dynamics in the next section.

To complete our results about density compaction, we have plotted in Fig. 4 the density profile $\rho(z)$ as a function of the height from the bottom of the box. Starting from a common initial configuration, the system is allowed to evolve while subjected to shaking with two different amplitudes $x=0.001$ and $x=0.1$. As already stated, this gives a difference in the two final density profiles, which seems to approximately have (see also [3]) a Fermi-Dirac dependence on the depth, z, as shown in Fig. 4:

$$\rho(z)=\rho_b\left[1-\frac{1}{1+e^{(z-z_0)/s}}\right], \tag{4}$$

where ρ_b is the asymptotic bulk density and z_0 and s are two fitting parameters describing the properties of the ''surface'' of the system. They all depend on the shaking amplitude x. The fit parameters for the initial profile are $\rho_b=0.53$, $z_0=35.3$, and $s=1.50$; after the shakes with $x=0.001$ we find $\rho_b=0.56$, $z_0=36.5$, and $s=0.83$, while after the shakes with $x=0.1$ we measure $\rho_b=0.575$, $z_0=37.1$, and $s=0.69$. All these show that, during compaction, the ''surface'' region of the system shrinks.

V. THE DENSITY-DENSITY AUTOCORRELATION FUNCTION

The above discussion about compaction under gentle shaking, with the presence of logarithmic relaxations, shows we are in the presence of a form of out-of-equilibrium process. To characterize its features in a quantitative way, we study the time-dependent correlation functions.

The system is allowed to evolve for a time interval t_w (the ''waiting time''), then correlations are measured as a function of time for $t>t_w$. In the present case, we record the relaxation features of the two-time density-density correlation function,

$$C(t,t_w)=\frac{\langle\rho(t)\rho(t_w)\rangle-\langle\rho(t)\rangle\langle\rho(t_w)\rangle}{\langle\rho(t_w)^2\rangle-\langle\rho(t_w)\rangle^2}, \tag{5}$$

where $\langle\cdots\rangle$ means the average over a number of random field configurations and different initial configurations. As above, $\rho(t)$ is the bulk density of the system at time t.

For a system at equilibrium the time-dependent correlation function $C(t,t_w)$ is invariant under time translations, i.e., it depends only on the difference $t-t_w$. However, if the

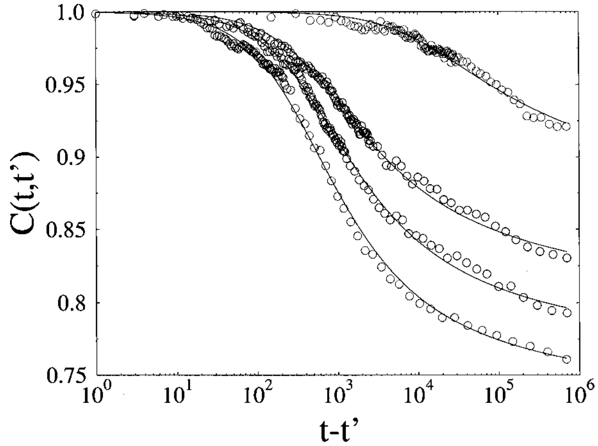

FIG. 5. The two-time density-density correlation function, $C(t,t_w)$, for a fixed vibration amplitude $x=10^{-2}$. The different curves correspond to four ''waiting times'' $t_w=72,360,720,7200$. $C(t,t_w)$ is not a simple function of $t-t_w$ and shows ''aging.''

system is off equilibrium, the subsequent response is expected to depend explicitly on the waiting time. Then $C(t,t_w)$ is a function of t and t_w separately. This ''memory'' effect is usually termed *aging* and plays an important role in the study of disordered thermal systems such as glassy systems [13].

As described above, the initial configuration of the system at $t=0$ is obtained by pouring grains in a cylindrical box from the top and letting them fall down randomly. In order to have the system in a well-definite configuration of its parameters, we started to shake it continuously with a fixed amplitude x, i.e., measures were taken during a single long ''tap.''

The results about the bulk density-density correlations of Eq. (5), averaged over 10 initial configurations and over 100 random field configurations, are shown in Fig. 5.

The various curves correspond to different waiting time values, $t_w=72,360,720,7200$ for a fixed shaking amplitude $x=0.01$. The dynamics of the system depends strongly on the value of the waiting time t_w, which is the signature of an aging phenomenon.

This behavior is not expected when shaking for long times at high x (i.e., high T), where our model is very close to a standard diluted lattice gas.

The specific scaling of the correlation function describes the system's aging properties. Therefore, it is interesting to analyze the features of $C(t,t_w)$ behavior as a function of t and t_w for small shaking amplitudes.

For long enough time, the correlation function scales with the ratio $\ln(t_w)/\ln(t)$. It can thus be approximated by a scaling form recently introduced to analyze memory effects in IFLG and TETRIS [10],

$$C(t,t_w)=(1-c_\infty)\frac{\ln\left(\dfrac{t_w+t_s}{\tau}\right)}{\ln\left(\dfrac{(t+t_s)}{\tau}\right)}+c_\infty, \tag{6}$$

where τ,t_s, and c_∞ are fitting parameters. It is worth noting that these fitting parameters, for a given x, seem to be constant for different waiting times, as shown in Fig. 6.

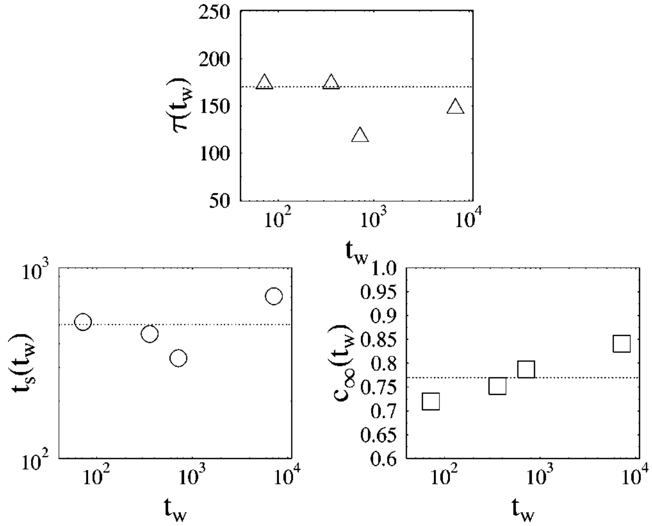

FIG. 6. The fitting parameters from Eq. (6) of the correlation function, $C(t,t_w)$, at $x=10^{-2}$, seem to be almost independent of the waiting time t_w.

All these results on time-dependent correlation functions thus confirm our picture of an off-equilibrium state of the dynamics.

VI. DISCUSSION

The above results thus show logarithmic scaling in off-equilibrium relaxations of a lattice-gas model with antiferromagnetic interactions and infinite random fields in the presence of gravity. These results are, interestingly, consistent with the known properties of standard random field Ising systems [12]. Surprisingly, they are also in strong correspondence with the behavior of analogous properties observed, by numerical investigation, in apparently different lattice models under gravity as the quoted TETRIS and IFLG, also introduced to describe granular media [10]. The TETRIS is a model which can be mapped, along the same lines outlined in the present paper, into a usual lattice gas in the presence of antiferromagnetic interactions of infinite strength and gravity. Its dynamics is ''frustrated'' by the presence of purely kinetic constraints: particles cannot turn their orientation if too many of their neighboring sites are filled. The IFLG is, instead, a model very close to an Ising spin glass (also with infinite interaction strengths). It may be described in terms of a lattice gas under gravity made of particles moving in an environment with quenched disorder. The presence of the quoted strong similarities in the off-equilibrium dynamics of these apparently heterogeneous systems suggests the existence of an unexpected inherent universality. This seems to be caused by the important effect of gravity on the ''frustrated'' dynamics of particles. The deep origin of such a phenomenon is still an open problem and it is an important issue to be further investigated.

VII. CONCLUSIONS

In conclusion, we have studied processes of density relaxation in the presence of gentle shaking in a lattice model for granular particles. The model has a simple geometrical interpretation in terms of elongated grains moving in a disordered environment and it admits a mapping into a random field Ising system with vacancies in the presence of gravity. The crucial ingredient in the model is the presence of geometric frustration dominating particle motion and the necessity of cooperative rearrangements. The present model shows logarithmic compaction of its bulk density and a two-time density-density correlation function, $C(t,t')$, which has an aging behavior well described by a logarithmic scaling $C(t,t')=\mathcal{C}(\ln(t')/\ln(t))$.

At this stage, an experimental check of our finding of aging in the process of granular media compaction will give a strong ground to our very simple model.

It is interesting that these results numerically coincide with the findings from two other kinds of lattice models which appear to be different: a model (the TETRIS) which can be mapped into an Ising antiferromagnetic lattice gas whose dynamics is characterized by purely kinetic constraints, and a model (the IFLG) which is, instead, closer to an Ising spin glass in the presence of gravity. The intriguing observation of these similarities seems to suggest the presence of a form of universality which appears in ‘‘frustrated’’ particle dynamics, due to the crucial effects of gravity.

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